COMPARISON BETWEEN TWO THEORETICAL FOOTPRINT MODELS AND EXPERIMENTAL MEASUREMENTS OF TURBULENT FLUXES: THE CASE STUDY OF LANDRIANO (PV)

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ABSTRACT

In this work two theoretical footprint models and experimental intra-field turbulent fluxes of latent, sensible heat and carbon dioxide are compared. Experimental data are obtained using a mobile eddy covariance station moving it from a discontinuity point, represented by the field edge, towards the centre of the field where a fixed eddy covariance station is placed. The experimental field is a bare soil at Landriano (PV) in the Po Valley, Italy. The experimental measurements show that latent and sensible heat fluxes have a hyperbolically trend across the field, while carbon dioxide flux is characterized by a linear trend. The maximum relative error between theoretical footprint models and turbulent scalar flux measurements is about 18%.

Keywords: eddy covariance, turbulent fluxes, footprint.

1 INTRODUCTION

The source area of a turbulent flux defines the spatial context of the measurement. It is something akin to the “field of view” of the measurement of surface atmosphere exchange. When turbulent flux sensors are deployed, the objective is usually to measure signals which reflect the influence of the underlying surface on turbulent exchange. The measured signal depends on which part of the surface has the strongest influence on the sensor, and thus on the location and size of its footprint (Schmid, 2002). The footprint size can be considered like a representative area of the sensor detector. Lots of models describe the representative source area for turbulent fluxes; Pasquill (1972) suggests an analogy between the developing zone of influence downwind from a surface element, and a diffusing plume of a scalar emitted at the surface element. He defines a surface area of influence, called “effective fetch”, bounded by a concentration isopleth of the plume, with an arbitrary value which is half of the maximum concentration. Dimensions of this quite elliptical fetch region, relative to sensor location, depend on sensor height, surface roughness and atmospheric stability conditions (Schmid, 2002). Gash (1985) adopts Pasquill’s (1972) idea to calculate a formula for the effective fetch of
micrometeorological evaporation measurements using diffusion theory Calder’s (1952) approximation for an uniform wind field and neutral conditions of the atmosphere. Schuepp et. al. (1990) is the first to coin the term “flux footprint”. They propose a differential footprint model for passive scalar flux under neutral conditions and they explore several approaches to the advection-diffusion equation to determine an approximate solution for the footprint area. Eulerian analytic models for flux footprint are subsequently developed by Horst & Weil (1992) who extend Schuepp et. al.’s (1990) results with more realistic and analytic dispersion model which takes into account atmospheric stability effects and wind speed variations with height. An alternative solution of the Eulerian advection-diffusion equation is the Lagrangian stochastic description of the trajectories of the passive particles in a turbulent flow (Schmid, 2002). Its application to footprint modeling assumes that the passive tracer dispersion can be represented by trajectories of a finite number of particles that are completely independent between each other. The trajectory or time evolution of the position depends on a first term which represents the deterministic part of the velocity increment, and the second term which represents the stochastic force (Thomson, 1978). A hybrid approach is shown in Hsieh et. al. (2000) which develops an approximate analytical model to estimate scalar flux footprint in thermally stratified atmospheric surface layer flows. The model is based on a combination of Lagrangian stochastic dispersion model results and dimensional analysis. The main advantage of this model is its ability to analytically combine atmospheric stability, measurement height, and surface roughness length with flux and footprint.

The eddy covariance technique is generally used to estimate turbulence fluxes of mass and energy from and towards surfaces. A difficult, but important issue remains to quantify the level of uncertainty due to flux measurements. In fact, these fluxes are complex processes, and their estimates come out from various measurements and calculations as well as numerous explicit and implicit assumptions (Wilson et. al., 2002). The eddy covariance technique is a statistical method which calculates flux as a covariance of instantaneous deviations in vertical wind speed and instantaneous deviations in the entity of interests (temperature, vapor concentration in the air, CO₂) (Burba & Anderson, 2010). Tridimensional sonic anemometer and gas analyzer are the instruments which in couple have to be used to measure the turbulent fluxes (Foken, 2008). Latent and sensible heat fluxes are directly estimated by the anemometer and gas analyzer which have to be located in a proper way into the field (Schmid, 2002), thus the evapotranspiration flux measurements are representative of the experimental field (Baldocchi & Rao, 1995). Micrometeorological station flux measurements are influenced by stability conditions of the atmosphere (Masseroni et. al., 2011; Foken, 2008) into the Atmospheric Boundary Layer (ABL) which represents the layer where the SVAT (Soil-Vegetation-ATmosphere) turbulent exchange takes place. A simple method that allows to understand if eddy covariance station works in a proper way is to compute the conservation energy; hence, the sum of turbulence fluxes of sensible and latent heat should balance the available energy.

In this work, Hsieh et. al. (2000) and Kormann & Meixner (2001) simple analytical footprint models, which describe the representative source area for turbulent fluxes, are compared with the experimental data.
2 THEORETICAL BACKGROUND

2.1 Hsieh Model

Hsieh et al. (2000) develops an approximate analytical model to estimate flux footprint in thermally stratified flows. This is a hybrid approach combining elements from Calder’s analytical solution (1952) with results of Thompson’s Lagrangian model (1987). In the analysis, they scaled Gash (1986) effective fetch with the Obukhov length and accounted for the stability effect introducing two similarity parameters $D$ and $P$, obtaining the Eq. (1).

$$
\frac{x}{L} = -\frac{1}{k^2 \ln(F/S_0)} D \left( \frac{z_u}{L} \right)^P
$$

Where $z_u$ is a length scale, function of the measurement height $z_m$ and surface roughness $z_0$. $k$ is the Von Karman constant, $L$ the Monin-Obukov length, $D$ and $P$ depends on stability conditions of the atmosphere (Hsieh et al., 2000). $F/S_0$ is the ratio between the measured flux by the sensor and the flux supposed to be emitted from the total field; the ratio range is always from 0 to 1 (Hsieh et. al., 2000). Finally, $x$ is the distance from the tower (fetch).

The footprint is expressed by Eq. (2).

$$
f(x, z_m) = \frac{1}{S_0} \frac{dF(x, z_m)}{dx} = \frac{1}{k^2 x^P} Dz_u^P \left| \frac{L}{x} \right|^{-P} e^{-\frac{1}{k^2 x^P} \frac{Dz_u^P}{L} \left| \frac{L}{x} \right|^{-P}}
$$

2.2 Kormann Model

The Kormann Model is based on a modification of the analytical solution of the advection-diffusion equation of Van Ulden (1978) and Horst & Weil (1979) for power law profiles of the mean wind velocity and the eddy diffusivity. To allow for the analytical treatment, the model assumes homogenous and stationary flow conditions over homogeneous terrain, it represents the vertical turbulent transport as a gradient diffusion process and it considers only the advection along wind direction. Assuming that vertical and crosswind dispersion are independent, the continuity equation reduces to a two-dimensional advection-diffusion equation.

In the Kormann Model the footprint is expressed as Eq. (3).

$$
f(x, z_m) = \frac{1}{x \Gamma \left( \frac{1 + m}{m} \right)} \left( \frac{z_u z_m^m}{r^2 \alpha_K x} \right)^{1+m} \exp \left( -\frac{z_u z_m^m}{r^2 \alpha_K x} \right)
$$

Where $\Gamma$ is the gamma function, $r$ the shape parameter described $r=2+m-n$ (Van Ulden 1978) and $\alpha_u, \alpha_K$ are proportionally constants determined by fitting the power laws for $u$ ($u=\alpha u z^m$) and $K$ ($K=\alpha K z^n$) to Monin-Obukov similarity theory (Foken, 2008).
STUDY SITE

The experiment is carried out in a maize field after reaping time at Landriano (PV) site in the Po Valley, Italy (45.19 N, 9.16 E, 87 m a.s.l.). The field, 10 hectare large, has a polygonal geometric structure and it is surrounded by rows plants which generate a significant discontinuity with the bare soil. A fixed eddy covariance tower (A) is installed more or less in the centre of the field (Figure 1a).

The station is equipped with the following sensors: one 3D sonic anemometer (Young 81000 by Young), which measures sonic temperature and the three components of wind speed and it is positioned at 5 m height; one gas analyzer (LICOR 7500 by LICOR) which stores water vapour and CO₂ concentartions at a frequency of 10 Hz (as well as the sonic anemometer) and it is positioned at 5 m height too, one radiometer (CNR1 by Kipp & Zonen), located at 4 m height, which measures the four components of net radiation, one thermo-hygrometer (HMP45C Campbell Scientific), located at 3.5 m, which measures air humidity and temperature; two thermocouples (by ELSI) and a heat flux plate (HFP01 by Hukseflux) for soil heat flux measurement are located at 6, 10 and 8 cm respectively; three humidity probes (CS616 by Campbell Scientific) for the volumetric soil moisture measurement at different depths , one rain gauge (AGR100 by Campbell Scientific) at 1.5 m height and finally one data logger CR5000 (Campbell Scientific) is used to store all data each 5 minutes during the experimental days.

Energy fluxes have been corrected applying Webb's correction for density fluctuations (Webb et. al., 1980) and the correction for buoyancy flux due to sonic temperature measurements (Liu et. al., 2001). Tilt correction has been applied to take into account that the assumption of negligible mean vertical velocity is not always verified (Tanner & Thurtell, 1969). Frequency response correction has been applied for the attenuation of eddy covariance fluxes due to sensor response, path-length averaging, sensor separation, signal processing, and flux averaging period (Massman & Lee 2002). Data during rainfall days are discarded. These corrections are applied to raw data using an automatic procedure PEC (Polimi Eddy Covariance) implemented by Politecnico of Milan (Corbari et al, 2012).

The experiment was carried out from the 258\textsuperscript{th} (14 September) to the 266\textsuperscript{th} (23 September) Julian day of the year 2011 and the database is composed by 2001 data. 700 data (about 27% out of total dataset) have a South-West wind direction.

The tower position in the field is compatible with the constant flux layer (CFL) (Elliot, 1958). CFL is defined as 10-15% of internal boundary layer (Savelyev & Taylor, 2005), and it represents a space area where measured fluxes by the eddy tower are constant. Following Elliot's (1958) formula, with a calculated aerodynamic roughness of about 0.041 m (Garrat, 1993), the CFL depth at the tower is about 6 meters ensuring that the eddy covariance instruments (tridimensional sonic anemometer and gas analyser) are included into the CFL.

In addition to the fixed tower, a mobile eddy covariance station (B) is used to carry out the experiment (Figure 1b). This station is mounted on a mobile extensible tripod where another tridimensional sonic anemometer (Young 81000 by Young) and a gas analyzer (LI-COR 7500 by LI-COR) are located at 5 m height. A CR5000 (by Campbell) data logger is used to store data every 5 minutes.
3.1 Experimental execution

To investigate the horizontal variation of the turbulent fluxes, the experiment is performed by placing the mobile eddy flux system (B) at various distances from the discontinuity line for extended periods (Figure 1c).

![Figure 1. a. Eddy covariance tower. b. Fixed tower (system A) and mobile station (system B). c. Map of the experimental site.](image)

The mobile system is placed at nominal distances of 0 (Point 1), 15 (Point 2) and 65 (Point 3) meters from the discontinuity point in a inclined straight line of about 191° in respect to North direction. The fixed tower is located at about 184 meters from discontinuity point.

In Point 3 the mobile station stayed from the 258th to the 263th Julian day, in Point 2 from the 263th to the 265th and in Point 1 from the 265th to the 266th. For this analysis, data are selected only when the wind direction is between 171° and 211°, as shown in Figure 1c, and the net radiation ($R_n$) was greater than 0 W m$^{-2}$. This assumption is associated to an unstable condition of the atmosphere which is the necessary condition to get correct measurements by eddy covariance stations (Masseroni et. al., 2011). To define different stability conditions of the atmosphere, the stability parameter $z_u/L$ (Hsieh et. al., 2000) is used. 1263 data are under unstable condition which is an important feature required by eddy covariance stations in order to measure the atmospheric turbulence. Convective conditions are mostly present during the daytime, while stable conditions are predominant during the night, when the atmospheric flows are quite laminar and the turbulence is small, therefore data in stable conditions have to be deleted.

4 RESULTS AND DISCUSSIONS

4.1 Measured fluxes across the field

Latent, sensible heat and CO$_2$ fluxes (F$_{CO2}$) are calculated using data acquired by system B for each measurement point and they are compared with measured data of the tower A. The slope of the linear regression ($m$) between measured fluxes of mobile and fixed station for different points from discontinuity line has been calculated, and the error of the linear regression curve in comparison with an ideal case 1:1 has been calculated as absolute value of the difference between 1 and $m$. This procedure allows to calculate the “Measured – cumulated footprint” as shown in Table 1.
For latent heat the spatial variability trend varies hyperbolically from the field edge in accordance with the literature experiment described in Baldocchi & Rao 1995. Sensible heat and carbon dioxide flux measurements are strongly connected with the characteristic of the soil (temperature, moisture, microbiological activities, ect). As shown in Horeschi (2008) the soil proprieties, assessed in Landriano site, have a large variability across the field and they should influence sensible heat and carbon dioxide fluxes measurements. As shown in Table 1, the sensible heat trend is constant from Point 1 to 2. Only in the Point 3 boundary condition effects tend to be negligible and the sensible heat flux is quite similar to the flux measurement in station A. The carbon dioxide flux trend is quite linear with the distance from the field edge. Unlike the latent and sensible heat, the carbon dioxide flux (in the Point 1) has been fixed to zero because the slope of the linear regression is negative, leading to a $F_{\text{CO}_2(B)/F_{\text{CO}_2(A)}}$ lower than zero.

In Table 1 normalized fluxes with respect to the fetch are also shown ("Measured-footprint-"). They define the contribution which is coming from the surface area at a certain distance away from instrument. Therefore, most of the contribution usually comes, not from underneath the instrument or from kilometres away, but rather from somewhere in between (Burba & Anderson, 2010). “Measured-footprint-” is the ratio between “Measured - cumulated footprint -” and the distance from the tower as explained in Hsieh et al. (2000).

For latent and sensible heat fluxes the maximum contribute is coming from Point 2 with a value of 0.049 m$^{-1}$ and 0.017 m$^{-1}$ respectively. For CO$_2$ flux the maximum contribute is at Point 3 with a value of flux equal to 0.058 m$^{-1}$.

### 4.2 Theoretical Model results

For each flux value measured by station A the footprint ($f$) and the cumulated footprint ($F/S_0$ ratio) has been calculated using relations described in Section 2. In theory, $f$ or $F/S_0$ are represented by curves modified in function of $x$ and $z_m$ (in this case $z_m$ for the station A and B is fixed to 5 m). For each input variable, the models produce footprint curves in function of $x$, where $x$ is a vector of the independent values. From the 258th to the 263th Julian day (Period 1) $x$ is set to 65 m, from the 263th to the 265th (Period 2) $x$ is set to 15 m and from the 265th to the 266th (Period 3) $x$ is set to 0 m to
obtain a single output value of $f$ and $F/S_0$. This procedure permits to obtain the footprint values that can be subsequently compared with the experimental results. The footprint value, obtained by setting the model with different $x$ values, is compared with the latent heat, sensible heat and carbon dioxide experimental measurements retrieved over the bare soil by station B at Point 1, 2 and 3 respectively. Following Hsieh et al. (2000), $f$ and $F/S_0$ (for $x$ equal to 0) are set to 0, while for $x$ equal to 184 m, $f$ and $F/S_0$ are set to 0 and 1 respectively. For each measured flux at the Period 1, 2 or 3, one value of $f$ and $F/S_0$ is calculated. Subsequently the mean and standard deviation for $F/S_0$ and $f$ values for each period and footprint model were calculated. In Table 2 results are summarized. The $F/S_0$ and $f$ standard deviation permits to understand the model reliability.

The $F/S_0$ and $f$ standard deviation are shown.

In Hsieh model the $F/S_0$ mean standard deviation is about 50% of the $F/S_0$ values while for $f$ the standard deviation is about equal to 22%. This model can be applied in all atmospheric stability conditions from convective to stable, but the simplification is due to the $D$ and $P$ parameters that derives by a regression of Eq. (1) to the results of the Thomson’s Lagrangian model (1987) as explain in Hsieh et al. (2000). The footprint curve peak location is at the Point 2, 15 m from the field edge, and the value of $f$ is about 0.021 m$^{-1}$.

In Kormann Model the $F/S_0$ mean standard deviation is about 52% of the $F/S_0$ values while for $f$ the standard deviation is about equal to 63%. The Kormann Model is constituted by $u$ and $K$ different parameterisations in related with the stability conditions of the atmosphere as show in Kormann & Meixner (2001). These parameterisations were calculated for the first time by Van Ulden (1978) but, over time, the experimental constants included in these formulas were changed as shown in Garrat (1993). In fact they are connected with the experimental field structure, homogeneity and atmospheric turbulence. As shown in Table 2, in reference to the Kormann Model, the peak location of the footprint curve is at the Point 2, and the value of $f$ is about 0.018 m$^{-1}$.

### Table 2. Theoretical footprint model results. Cumulated flux footprint, footprint and standard deviation are shown.

<table>
<thead>
<tr>
<th>Distance from the field edge (m)</th>
<th>Hsieh Model</th>
<th>Kormann Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_0$</td>
<td>$F_0$</td>
</tr>
<tr>
<td></td>
<td>$F_0$</td>
<td>$F_0$</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>15</td>
<td>0.26</td>
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</tr>
<tr>
<td>65</td>
<td>0.65</td>
<td>0.004</td>
</tr>
<tr>
<td>184</td>
<td>1.00</td>
<td>0.000</td>
</tr>
</tbody>
</table>

|                                 | $f$ (1 m$^{-1}$) | $f$ (1 m$^{-1}$) |
|                                 | $f$ (1 m$^{-1}$) | $f$ (1 m$^{-1}$) |
| 0                               | 0.000          | 0.000           |
| 15                              | 0.16           | 0.26            |
| 65                              | 0.18           | 0.33            |
| 184                             | 0.021          | 0.003           |

|                                 | $f$ (1 m$^{-1}$) | $f$ (1 m$^{-1}$) |
|                                 | $f$ (1 m$^{-1}$) | $f$ (1 m$^{-1}$) |
| 0                               | 0.000          | 0.000           |
| 15                              | 0.018          | 0.011           |
| 65                              | 0.004          | 0.002           |
| 184                             | 0.000          | 0.000           |

4.3 Experimental data compared with footprint model results

In Figure 2 the variation of latent, sensible and carbon dioxide fluxes across the field and comparison with theoretical footprint curves is shown.
Figure 2. Variation of the latent, sensible, carbon dioxide fluxes with distance from the field edge and comparison with theoretical models (top). Comparison between measured and model predicted footprint ($F/SA$) (bottom). The 1:1 line is also shown.

At this site, the surface flux upwind is not zero and the source strength is simply approximated as Eq. (4).

$$ S(x) = \begin{cases} S_1 & \text{for } x < 0 \\ S_A & \text{for } x \geq 0 \end{cases} \quad (4) $$

Where the leading edge is at $x=0$, $S_1$ and $S_A$ are determined from the measured fluxes at $x=0$ m and $x=184$ m, respectively. By superposition, it is possible to calculate the flux using the relation described in Hsieh et al. (2000) (Eq. (5)).

$$ F(x, z_m) = S_1 \int_{-\infty}^{x} f(x, z_m) dx + S_A \int_{0}^{x} f(x, z_m) dx \quad (5) $$

In Eq. (5) for $x \rightarrow 0$, $F(x, z_m) \rightarrow S_1$; for $x \rightarrow \infty$, $F(x, z_m) \rightarrow S_A$. This is due to the effect of boundary conditions. At the field edge (Point 1), the mobile station B, which has its footprint, is influenced by the upwind flux. The $F/S_A$ ratio is 0.45 for latent heat, 0.26 for sensible heat and 0 for carbon dioxide. The upwind flux effect tends to be negligible when the mobile station B is moved through the field, and the Eq. (5) permits to take into account this phenomenon. Figure 2 (top) shows the variation of latent, sensible and carbon dioxide fluxes with fetch (downwind distance) above bare soil; solid lines depicts the proposed model predictions and the dots depicts eddy covariance measurements. For latent and sensible heat the predicted variation obtained by Kormann Model is compared reasonably well with observed data. To further examine the model performance, theoretical model predictions were compared with experimental results in the scatter plot shown in Fig. 2 (bottom). In Table 3, $m$ and $R^2$ for each model are summarized and the standard error is shown. For latent and sensible heat flux, the
Comparison between theoretical footprint models and experimental turbulent flux measurements

Kormann Model shows a good agreement with measurements and the standard error, calculated as one complement to the 1:1 ideal case, is 0.005 and 0.025 respectively.

<table>
<thead>
<tr>
<th>Model</th>
<th>Latent Heat (LE)</th>
<th>Sensible Heat (H)</th>
<th>Carbon Dioxide (F CO₂)</th>
<th>ErrLE</th>
<th>ErrH</th>
<th>ErrFCO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m</td>
<td>R²</td>
<td>m</td>
<td>R²</td>
<td>(-)</td>
<td>(-)</td>
</tr>
<tr>
<td>Hsieh</td>
<td>0.937</td>
<td>0.925</td>
<td>0.964</td>
<td>0.846</td>
<td>1.129</td>
<td>0.063</td>
</tr>
<tr>
<td>Kormann</td>
<td>0.995</td>
<td>0.997</td>
<td>1.025</td>
<td>0.616</td>
<td>1.180</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 3. Slope of linear regression, R² and percentage relative errors of the modeled footprint results in comparison with measured experimental data.

5 CONCLUSION

A field study was conducted to investigate the variability of turbulent mass and energy fluxes across a bare soil located at Landriano in the Po Valley.

Latent, sensible heat and CO₂ fluxes measured at 5 m above the surface showed significant horizontal variations with downwind distance from discontinuity line. Latent and sensible heat cumulative fluxes are quite similar. The magnitude of LE and H increases rapidly with the distance over the bare soil. CO₂ measured fluxes trend is totally different by LE and H.

Three different footprint model types are compared with experimental results showing that the Kormann Model is in good agreement with latent and sensible heat fluxes measurements.

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REFERENCES


Van Ulden A. Simple estimates for vertical diffusion from sources near the ground. *Atmospheric Enviroment.* 1978, 12, 2125-2129.
