IDRA 2010

XXXII Convegno Nazionale di Idraulica e Costruzioni Idrauliche

XXXII Italian Conference of Hydraulics and Hydraulic Construction

Palermo, 14 – 17 Settembre 2010

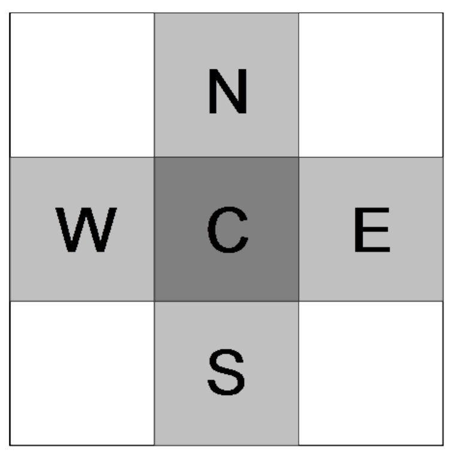
A SIMPLE GROUNDWATER MODEL BASED ON CELLULAR AUTOMATA PARADIGM

G. Ravazzani(1), D. Rametta(2) and M. Mancini(1)
(1) Politecnico di Milano, Italy – giovanni.ravazzani@polimi.it
(2) ENI spa, Italy

MODEL DESCRIPTION

MACROSCOPIC CELLULAR AUTOMATA

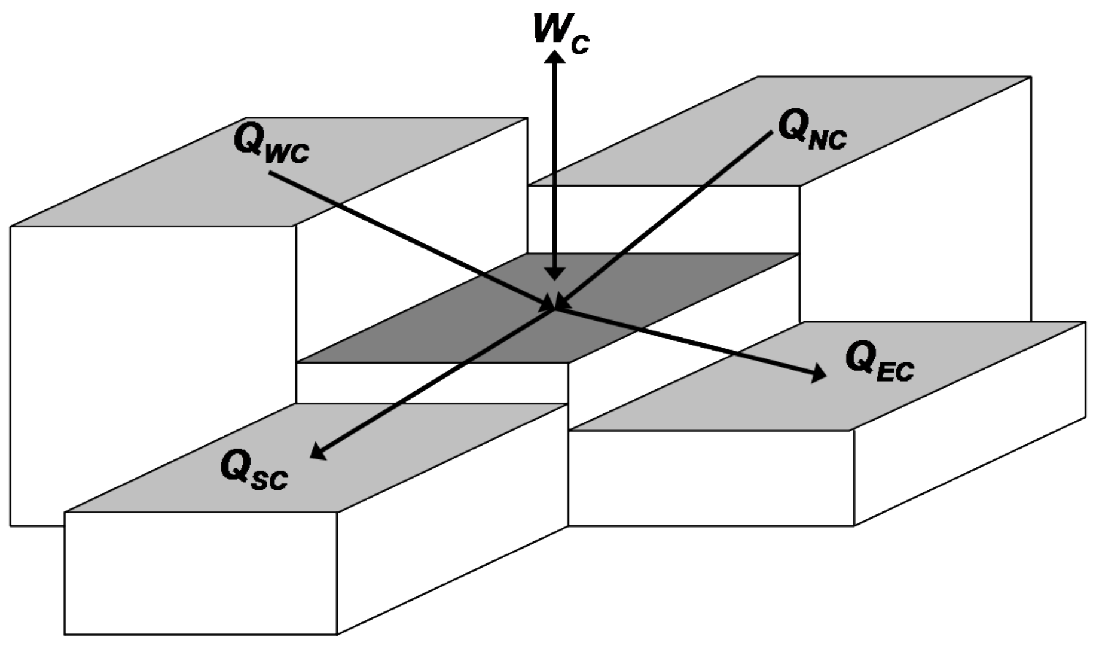
Models based on CA paradigm consist of four primary components: a **lattice** of cells, the definition of a **local neighbourhood** area, **transition rules** determining the changes in cell properties, and **boundary conditions**



LOCAL NEIGHBOROUD: VON NEUMANN APPROACH

MACCA-GW

MACroscopic **C**ellular **A**utomata for **G**round**W**ater modelling



TRANSITION RULES

$$Q_{NC} = \frac{2T_N T_C}{T_N + T_C}\left(h_N^t - h_C^t\right)$$

Darcy law (physical meaning)

$$Q_C = Q_{NC} + Q_{EC} + Q_{SC} + Q_{WC} + W_C$$

Total flux entering the central cell

$$h_C^{t+1} = h_C^t + \frac{1}{S_y}\frac{Q_C}{\Delta s^2}\Delta t$$

Hydraulic head at central cell is updated

BOUNDARY CONDITIONS

Dirichlet conditions: specified head, *h*

Neumann conditions: specified head gradient, $\partial h/\partial x$

type A (permeable), $\partial h/\partial x \neq 0$

type B (impermeable), $\partial h/\partial x = 0$

ABSTRACT: A groundwater model representing two-dimensional flow in unconfined aquifers is presented. The model is based on the paradigm of the macroscopic cellular automata, that represents dynamical systems which are discrete in space and time, operate on a uniform, regular lattice and are characterised by local interactions. Physically based equations are implemented to simulate the flow of water between adjacent cells. The model was validated against solutions of simple problems both in steady and transient condition including analytical solution and simulation performed with MODFLOW-2000 model. The developed code is simple enough to facilitate its integration into other models such as land surface models.. The good performance without detriment to accuracy makes the model adequate to perform long simulation time analysis.

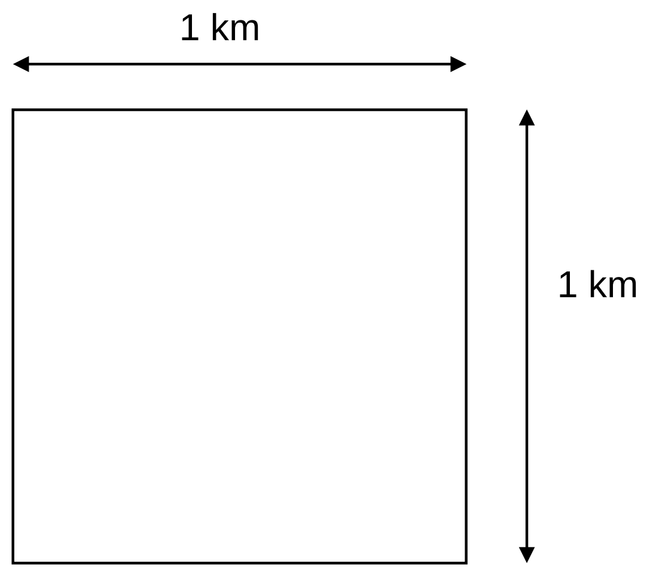
MODEL PERFORMANCE

Summary of the results of the computational performance analysis performed with different specific yield (Sy) and cell Reynolds Number (D). Table shows calculating time, Tcalc, in seconds, and root mean square error at monitoring well at 127 m distance in 45 degrees direction (RMSE W =127) and 200 m distance in cardinal direction (RMSE W =200) from the pumping well.

Case	*S_y (-)*	*Δt (s)*	*D*	*Model*	*T_{calc} (s)*	*RMSE W=127 (m)*	*RMSE W=200 (m)*
1	0.1	4000	1	MACCA-GW	1.125	9.54E-05	2.72E-05
				MODFLOW	5.204	2.35E-04	1.39E-04
2	0.1	16000	4	MODFLOW	1.406	2.51E-04	1.64E-04
3	0.1	32400	8.1	MODFLOW	0.812	6.50E-04	2.66E-04
4	0.1	64800	16.2	MODFLOW	0.531	1.04E-03	4.66E-04
5	0.3	12000	1	MACCA-GW	0.375	1.95E-04	2.07E-05
				MODFLOW	2.703	7.86E-05	5.28E-05
6	0.3	48000	4	MODFLOW	0.562	2.44E-04	9.01E-05
7	0.3	99692	8.3	MODFLOW	0.375	5.33E-04	1.41E-04
8	0.3	185143	15.4	MODFLOW	0.235	1.00E-03	3.05E-04

MODEL TESTING

ARTIFICIAL TEST CASE



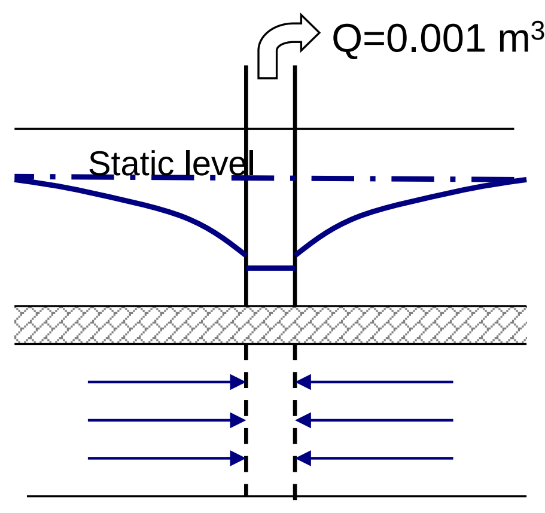
Ks = 1.25·10⁻⁵ m/s

S_y = 0.1

ΔS = 10 m

Grid nodes = 100 x 100 = 10000

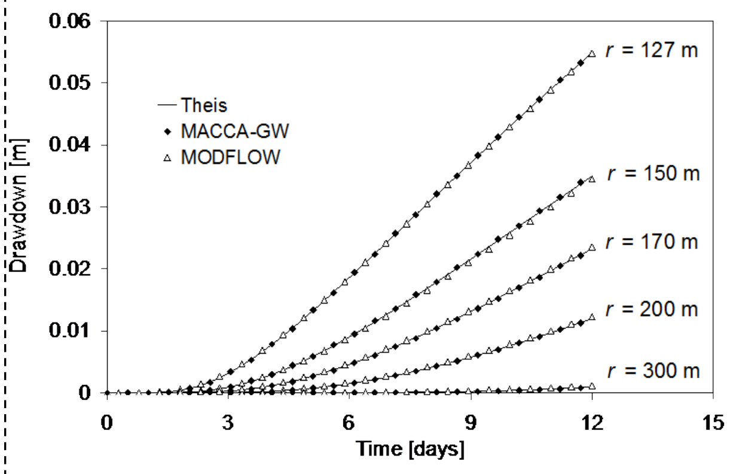
GROUNDWATER PUMPING

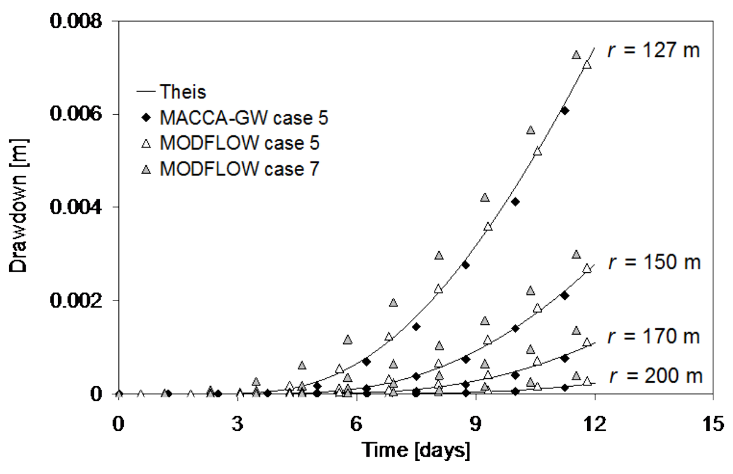


Theis (1935): $s(r,t) = \frac{Q}{4\pi T}\cdot W(u)$ $\begin{cases} u = \dfrac{r^2 \cdot S}{4tT} \\ W(u) = \displaystyle\int_u^\infty \frac{e^{-z}}{z} \end{cases}$

Δt = 4000 s

Layer thickness = 50 m





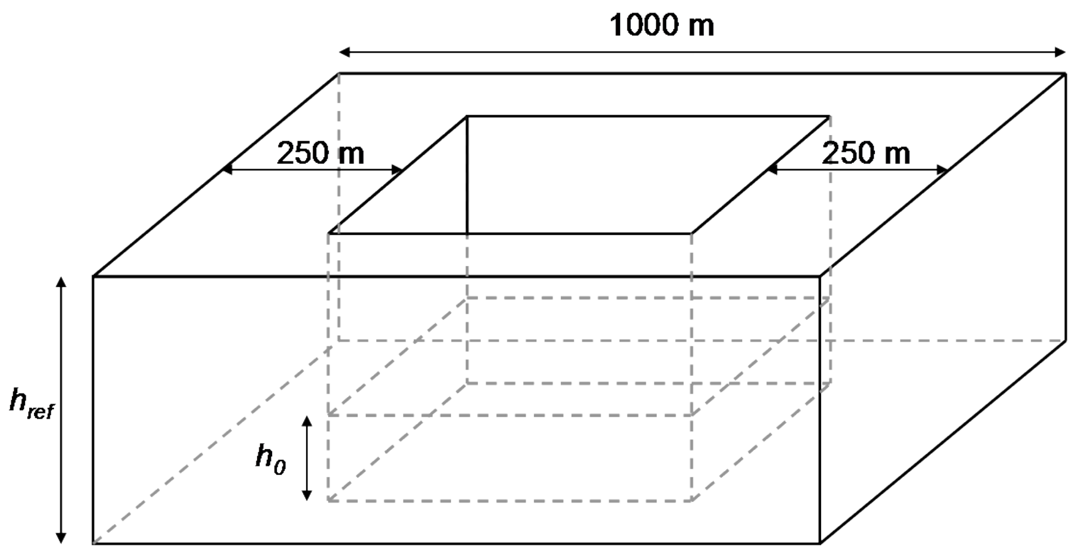
CONVERGENCE TEST

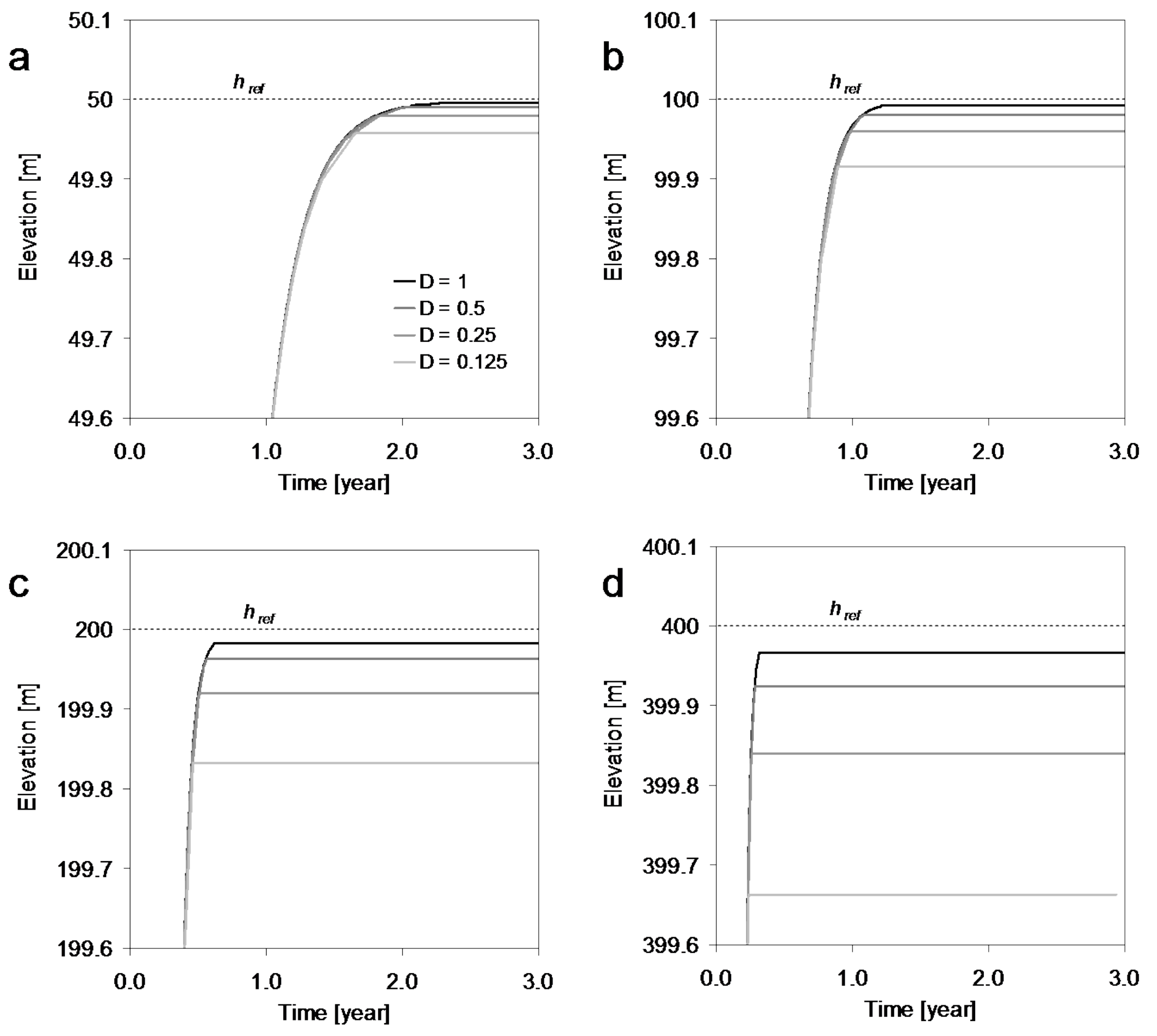
$$\upsilon = \frac{T}{S}$$

Hydraulic diffusivity

$$D = \frac{\upsilon}{\frac{(\Delta s/2)^2}{\Delta t}} = 4\upsilon\frac{\Delta t}{(\Delta s)^2}$$

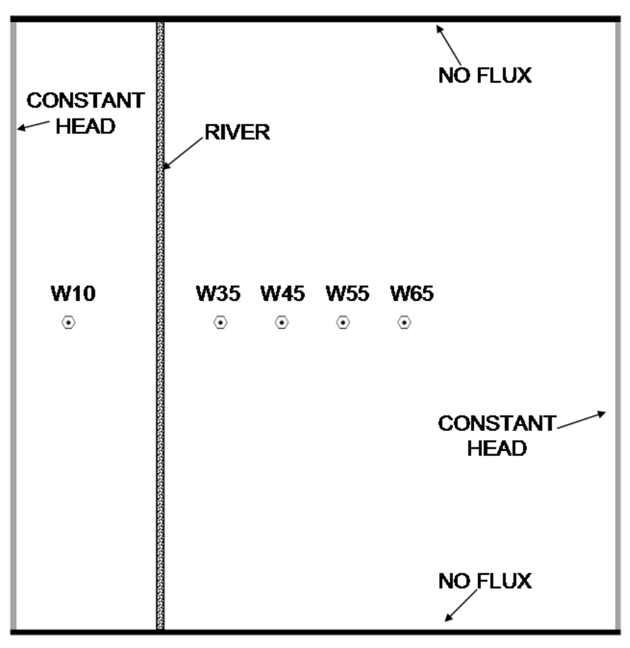
Cell Reynolds number(*)

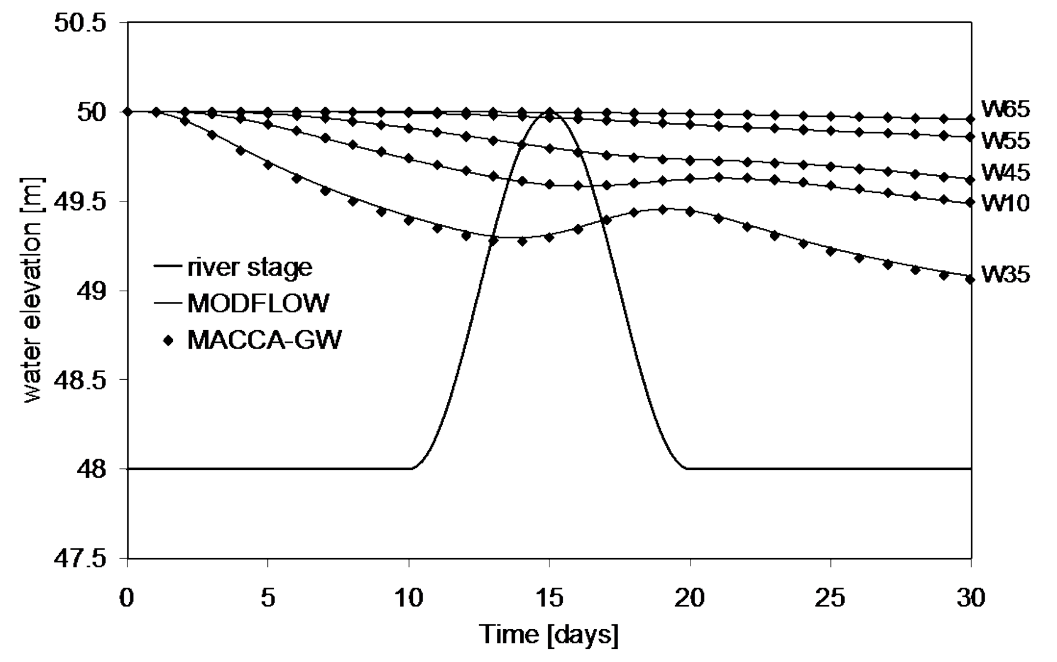


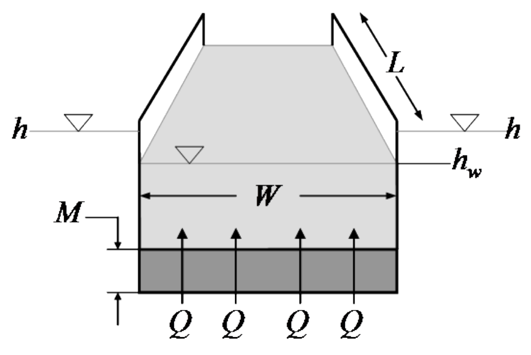


(*) Ponce, V.M., Shetty, A.V., Ercan, S., 2001. A convergent explicit groundwater model. Webpublished. http://attila.sdsu.edu/~ponce/gwmodelpaper.html.

STREAM STAGE VARIATION







$$Q = \frac{K_{sb}LW}{M}\left(h_w - h\right)$$

CONCLUSIONS

A cellular automata on a regular grid representing two-dimensional groundwater flow in unconfined aquifer was presented. Physically based equations are implemented to simulate the flow of water between adjacent cells. This makes easier the setting of model parameters and their calibration. The model can account for sources or sinks and boundary conditions of Dirichlet or Neumann type. River-aquifer interaction can be simulated: the stream stage is used to calculate the flux between the stream and aquifer system, proportional to the head gradient between the river and aquifer and a streambed conductance parameter.

Test of the model under hypothetical conditions showed that the model is stable and convergent when the time step satisfies the condition that cell Reynolds number D = 1.

The accuracy of the model was evaluated considering three testing problems both in transient and steady state: the steady flow between two streams in response to uniform recharge, the drawdown due to a constant pumping rate from a well, and the aquifer response to stream-stage variation. Comparison with analytical solution and MODFLOW-2000 numerical results showed a good agreement.

The MACCA-GW model, thank to the explicit numerical scheme based on macroscopic cellular automata that does not perform inner iterations, proved to be fast in simulating the investigated transient phenomena. Simulations were performed investigating drawdown due to a constant pumping rate from a well with different values of specific yield and time step. For cell Reynolds number, D = 1, MACCA-GW generally exhibited more accuracy and resulted from 4.6 to 7.2 times faster than MODFLOW-2000. For D > 1, with the increase of time step, MACCA-GW showed instability and MODFLOW computational time decreased but RMSE increased of nearly two orders of magnitude.

The code of MACCA-GW model is simple enough to facilitate its integration into other models such as distributed model that simulate water and energy fluxes at the interface between soil and atmosphere. The good performance in terms of calculating time without detriment to model's accuracy, makes the MACCA-GW adequate to perform long simulation time analysis.

Acknowledgements: The work was supported in the framework of the ACQWA EU/FP7 project (grant number 212250) "Assessing Climate impacts on the Quantity and quality of WAter".