Macroscopic Cellular Automata for groundwater modelling

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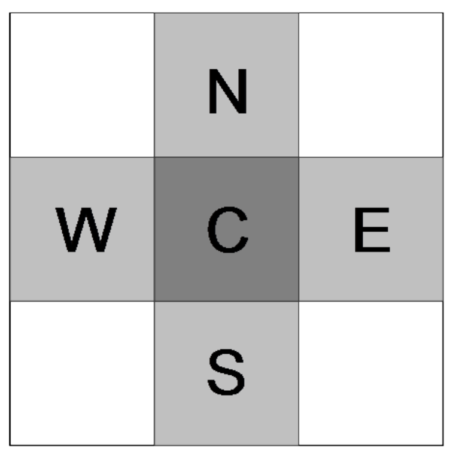
MODEL DESCRIPTION

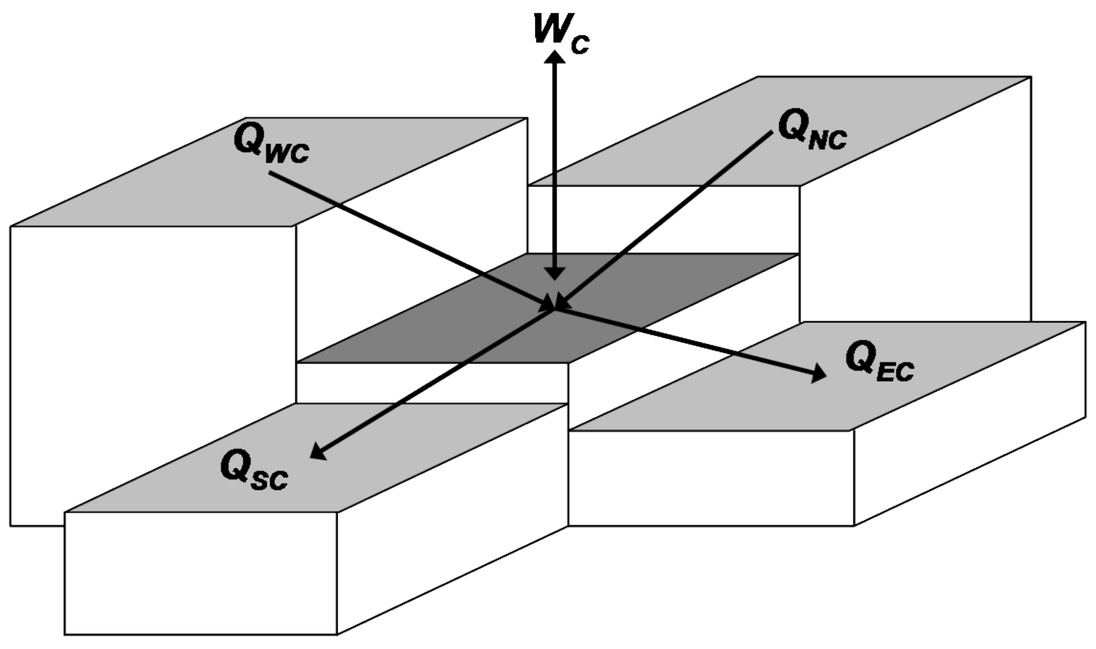
MACROSCOPIC CELLULAR AUTOMATA

Models based on CA paradigm consist of four primary components: a **lattice** of cells, the definition of a **local neighbourhood** area, **transition rules** determining the changes in cell properties, and **boundary conditions**



LOCAL NEIGHBOROUD:
VON NEUMANN APPROACH

TRANSITION RULES



$$Q_{NC} = \frac{2T_N T_C}{T_N + T_C}\left(h_N^t - h_C^t\right)$$

Darcy law (physical meaning)

$$Q_C = Q_{NC} + Q_{EC} + Q_{SC} + Q_{WC} + W_C$$

Total flux entering the central cell

$$h_C^{t+1} = h_C^t + \frac{1}{S_y}\frac{Q_C}{\Delta s^2}\Delta t$$

Hydraulic head at central cell is updated

BOUNDARY CONDITIONS

Dirichlet conditions: specified head, *h*

Neumann conditions: specified head gradient, $\partial h/\partial x$

→ type A (permeable), $\partial h/\partial x \neq 0$

→ type B (impermeable), $\partial h/\partial x = 0$

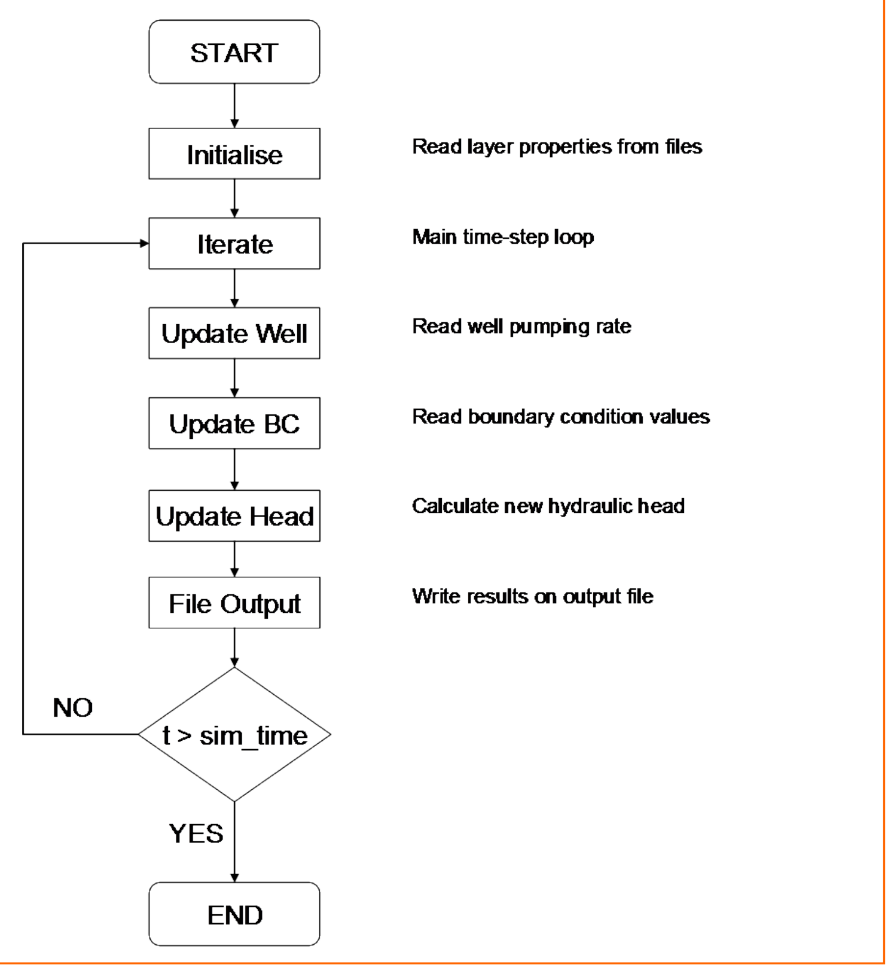
ABSTRACT: A groundwater model representing two-dimensional flow in unconfined aquifers is presented. The model is based on the paradigm of the macroscopic cellular automata, that represents dynamical systems which are discrete in space and time, operate on a uniform, regular lattice and are characterised by local interactions. Physically based equations are implemented to simulate the flow of water between adjacent cells. The model was validated against solutions of simple problems both in steady and transient condition including analytical solution and simulation performed with MODFLOW-2000 model. The developed code is simple enough to facilitate its integration into other models such as land surface models.. The good performance without detriment to accuracy makes the model adequate to perform long simulation time analysis.

MODEL IMPLEMENTATION

MACCA-GW MACroscopic Cellular Automata for GroundWater modelling

Fortran 90

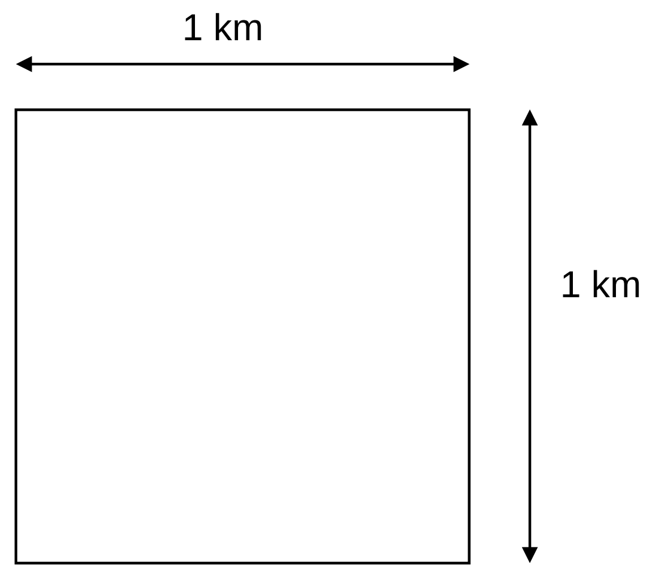
```
TYPE grid_integer
        INTEGER,POINTER :: mat (:,:)     !grid data
        INTEGER         :: jdim          !number of columns
        INTEGER         :: idim          !number of rows
        REAL            :: xllcorner     !lower left corner x coordinate
        REAL            :: yllcorner     !lowerleft corner y coordinate
        REAL            :: cellsize      !cell dimension
        INTEGER         :: nodata        !nodata value
END TYPE grid_integer

TYPE grid_real
        REAL,POINTER    :: mat (:,:)     !grid data
        INTEGER         :: jdim          !number of columns
        INTEGER         :: idim          !number of rows
        REAL            :: xllcorner     !lower left corner x coordinate
        REAL            :: yllcorner     !lowerleft corner y coordinate
        REAL            :: cellsize      !cell dimension
        REAL            :: nodata        !nodata value
END TYPE grid_real

TYPE layer
    TYPE (grid_real)        :: top           !top elevation
    TYPE (grid_real)        :: bottom        !bottom elevation
    TYPE (grid_real)        :: KsLayer       !layer hydraulic conductivity
    TYPE (grid_real)        :: porosity      !effective porosity
    TYPE (grid_real)        :: KsAquitard    !aquitard hydraulic conductivity
    TYPE (grid_integer)     :: domain        !domain of analysis and
                                             !boundary condition type
    TYPE (grid_real)        :: bc            !boundary condition values
END TYPE layer
```



Read layer properties from files
Main time-step loop
Read well pumping rate
Read boundary condition values
Calculate new hydraulic head
Write results on output file

MODEL TESTING

ARTIFICIAL TEST CASE



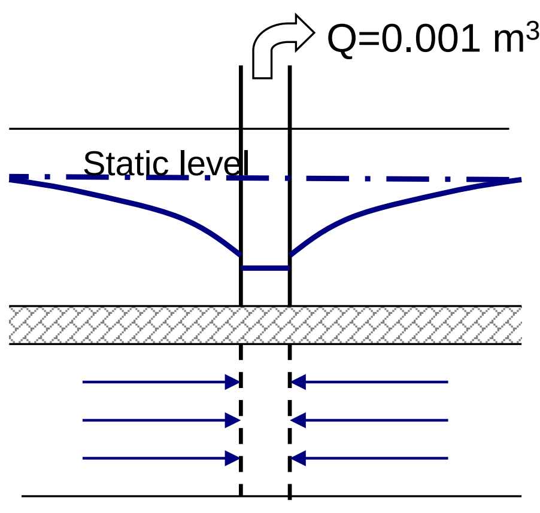
Ks = 1.25·10^{-5} m/s

S_y = 0.1

ΔS = 10 m

Grid nodes = 100 x 100 = 10000

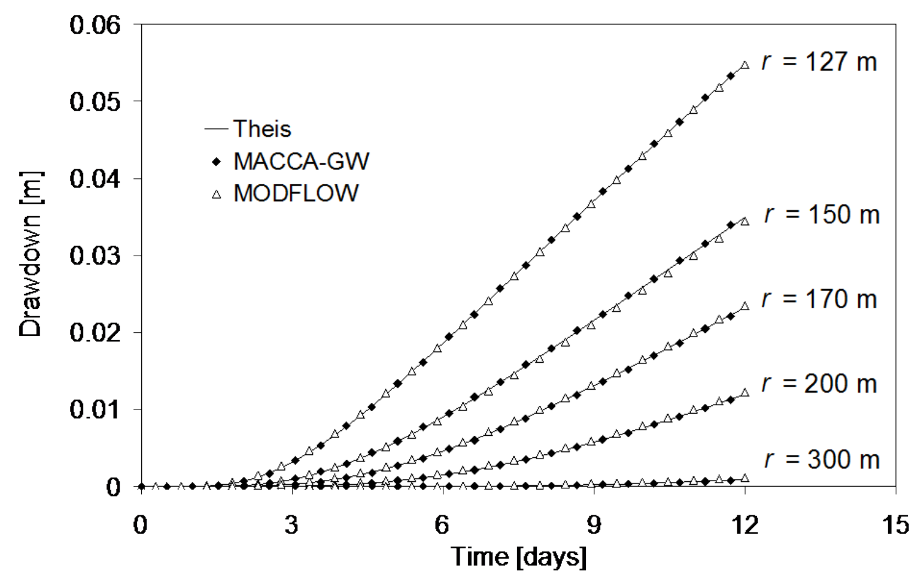
GROUNDWATER PUMPING



Theis (1935): $s(r,t) = \frac{Q}{4\pi T}\cdot W(u)$ $\begin{cases} u = \dfrac{r^2 \cdot S}{4tT} \\ W(u) = \displaystyle\int_u^\infty \dfrac{e^{-z}}{z} \end{cases}$

Δt = 4000 s

Layer thickness = 50 m



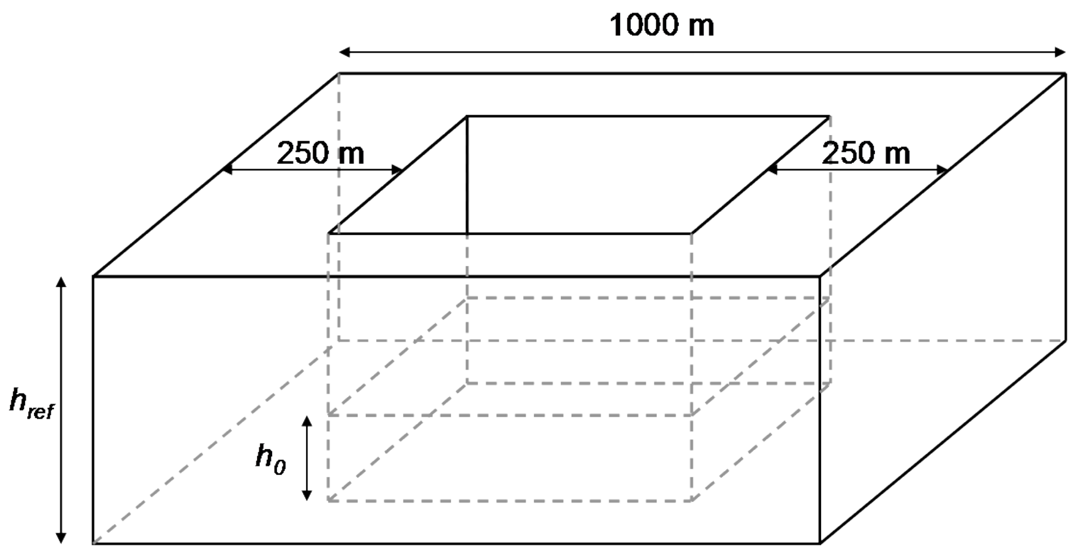
CONVERGENCE TEST

$$\upsilon = \frac{T}{S}$$

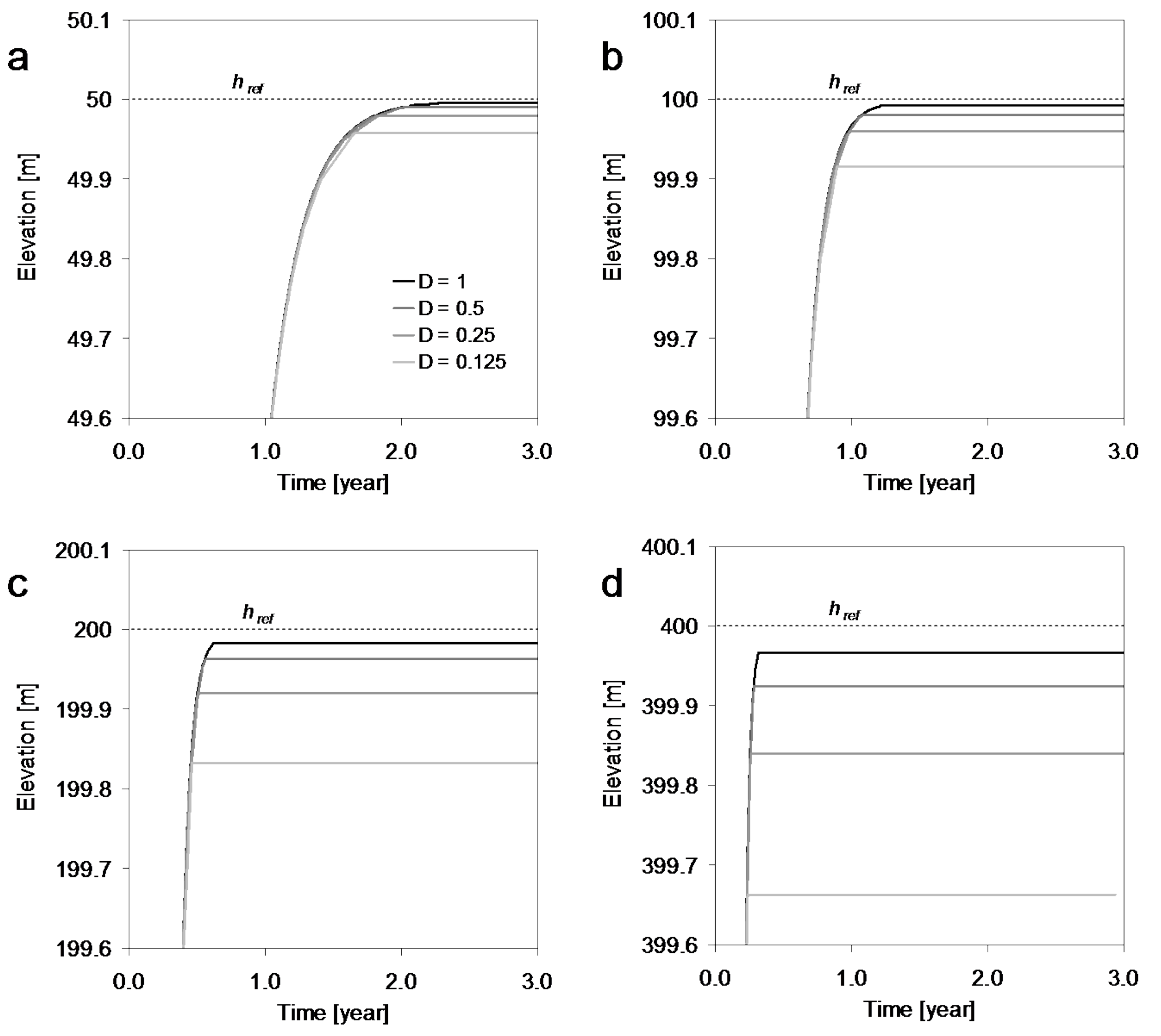
Hydraulic diffusivity

$$D = \frac{\upsilon}{\frac{(\Delta s/2)^2}{\Delta t}} = 4\upsilon\frac{\Delta t}{(\Delta s)^2}$$

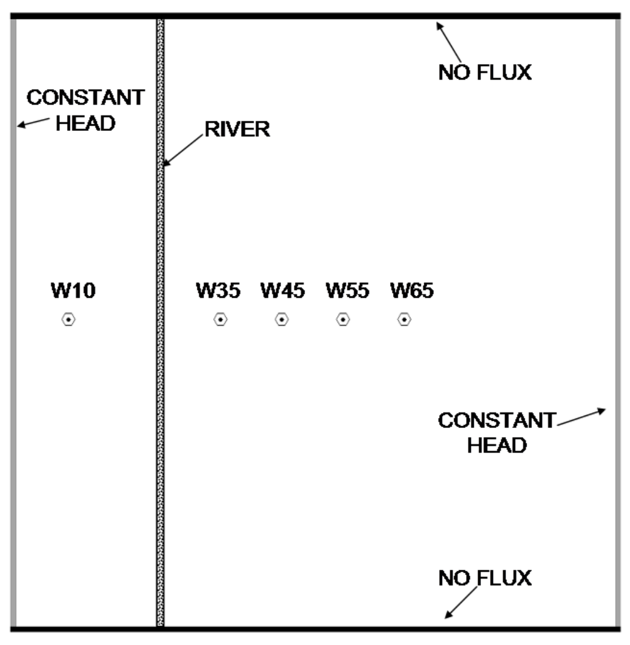
Cell Reynolds number(*)

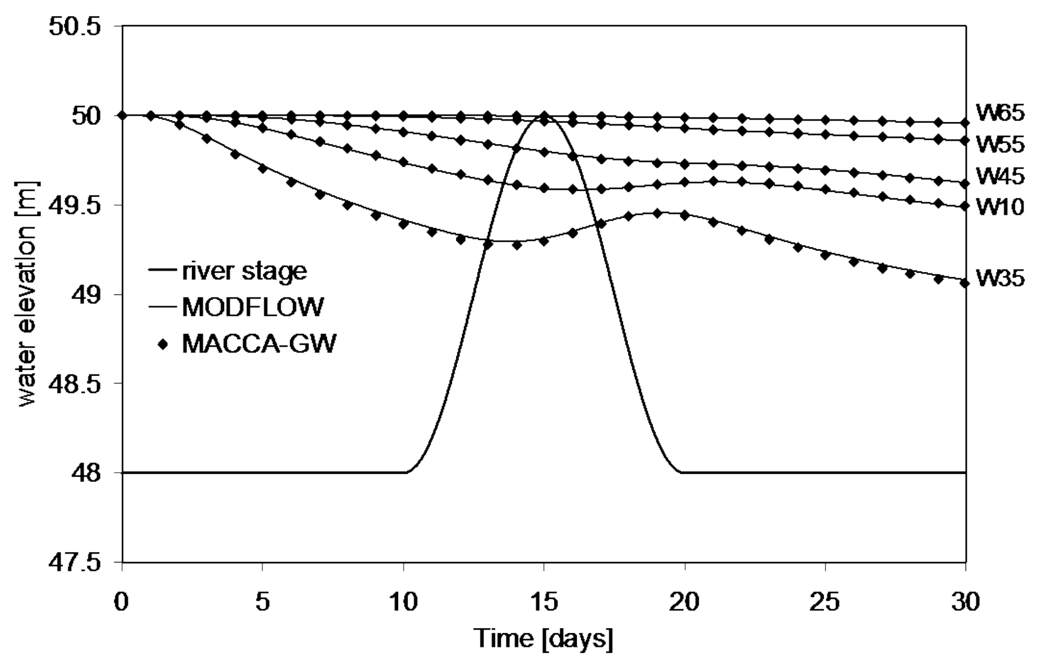


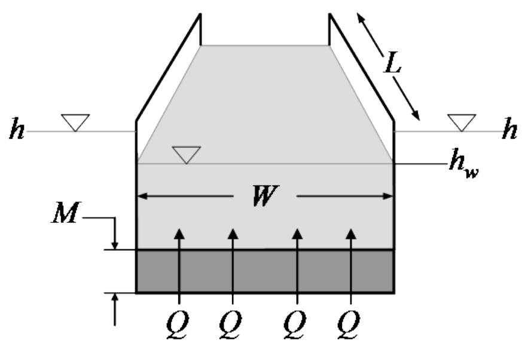
(*) *Ponce, V.M., Shetty, A.V., Ercan, S., 2001. A convergent explicit groundwater model. Webpublished. http://attila.sdsu.edu/~ponce/gwmodelpaper.html.*



STREAM STAGE VARIATION







$$Q = \frac{K_{sb}LW}{M}\left(h_w - h\right)$$

CONCLUSIONS

Test of the model under hypothetical conditions showed that the model is stable and convergent when the time step satisfies the condition that cell Reynolds number D = 1. The accuracy of the model was evaluated considering three testing problems both in transient and steady state: the steady flow between two streams in response to uniform recharge, the drawdown due to a constant pumping rate from a well, and the aquifer response to stream-stage variation. Comparison with analytical solution and MODFLOW-2000 numerical results showed a good agreement.

The MACCA-GW model, thank to the explicit numerical scheme based on macroscopic cellular automata that does not perform inner iterations, proved to be fast in simulating the investigated transient phenomena: it resulted from 4.6 to 12 times faster than MODFLOW-2000.

The code of MACCA-GW model is simple enough to facilitate its integration into other models such as distributed model that simulate water and energy fluxes at the interface between soil and atmosphere. The good performance in terms of calculating time without detriment to model’s accuracy, makes the MACCA-GW adequate to perform long simulation time analysis.

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