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A SIMPLE GROUNDWATER MODEL BASED ON CELLULAR AUTOMATA PARADIGM

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ABSTRACT

A groundwater model representing two-dimensional flow in unconfined aquifers is presented. The model is based on the paradigm of the macroscopic cellular automata, that represents dynamical systems which are discrete in space and time, operate on a uniform, regular lattice and are characterised by local interactions. Physically based equations are implemented to simulate the flow of water between adjacent cells. The model was validated against solutions of simple problems including analytical solution and simulation performed with MODFLOW-2000 model. The developed code is simple enough to facilitate its integration into other models such as land surface models.. The good performance without detriment to accuracy makes the model adequate to perform long simulation time analysis.

1 INTRODUCTION

Groundwater models historically have been applied to aquifer management problems (*Ponzini et al.*, 1989). To this purpose, many sophisticated numerical models have been developed to simulate water fluxes in complex heterogeneous multi layered aquifers (*Zyvoloski*, 2007; *Hughes and Liu*, 2008), while surface water processes are often oversimplified ignoring runoff, actual evapotranspiration, and snow dynamics (*Giudici et al*, 2000). On the other hand, traditional Land-Surface Models (LSM) are designed with emphasis on surface water movement whereas the subsurface is commonly simulated by means of simple conceptual approaches or assumed as zero flux boundary (*Niu et al.*, 2007). However, in many circumstances the interaction between surface and groundwater plays a crucial role, so that integrated modelling approaches become fundamental for application to water resources planning and management (*Facchi et al.*, 2004), also in light of the demands of the European Water Framework Directive (WFD; 2000/60/EU). In these situations, despite the eventual complexity of the aquifer system, the modelling of the surficial unconfined layer, is often sufficient to simulate water exchange between surface water and the underlying water table (*Wondzell et al.*, 2009). Moreover, in order to run long time simulations at fine resolution, the model is required to be as simple as possible to provide reliability, efficiency and flexibility. Fortunately, the common belief that very complex phenomena require necessarily sophisticated models has been shown to be erroneous: complexity can arise in a model even if governed by very simple rules (cf. e.g. *Wolfram*, 2002). Among these approaches Cellular Automata (CA) represent a simple, attractive and alternative modelling technique respect to traditional numerical models that solve differential equations to describe complex phenomena (*Toffoli,* 1984). Cellular Automata are dynamical systems which are discrete in space and time, operate on a uniform, regular lattice and are characterised by local interactions. They were introduced by *von Neumann* (1966) to study self-reproducing systems and have been later used for modelling disparate complex physical phenomena (*Di Gregorio et al.*, 1999). Models based on CA are directly compatible with parallel programming and so they allow to easily exploit the power of modern computers.

These CA make use of local laws that are ruled by empirical parameters. As these latter can have no direct link with classical physical parameters, an accurate calibration phase is generally required (*Iovine et al.*, 2005). At the contrary, physically based Macroscopic Cellular Automata (MCA), in which local rules derive directly by physical laws and depend on physical parameters, do not require a similar calibration (*Mendicino et al.*, 2006).

In this work, physically based MCA are the reference computational paradigm of a new two-dimensional model developed to simulate water flux in saturated aquifers. The model is developed for its inclusion in a distributed hydrological model (*Ravazzani et al.*, 2007; *Rabuffetti et al.*, 2008), with the aim of simulating water exchange between surface soil, river network and the underlying aquifer. The model is validated against typical problems in the study of alluvial aquifers: transient drawdown due to a constant pumping rate from a well, and aquifer response to stream-stage variation. Benchmarks include analytical solution and numerical simulation performed with MODFLOW-2000.

2 MODEL FORMULATION

The developed model, MACCA-GW (MACroscopic Cellular Automata for GroundWater modelling), is based on CA paradigm and consists of four primary components: a lattice of cells, the definition of a local neighbourhood area, transition rules determining the changes in cell properties, and boundary conditions (*Parsons & Fonstad*, 2007). To simulate water flux in unconfined aquifer, a two-dimensional lattice of cells is created. Each cell is instantiated with a value of saturated hydraulic conductivity, K_s [L/T], a value of specific yield, S_y [-], elevation of the bottom of the aquifer [L] and initial head [L]. The cell size must be small enough so that physical properties can be considered homogeneous in the cell space, but large enough to achieve macroscopic description of the physical processes. The cell size is set as $\Delta s = \Delta x = \Delta y$.

The neighbourhood in CA models defines the area of process influence. Among those proposed in literature for two-dimensional CA with square tessellation, as that here presented, the von Neumann and Moore ones are the most adopted: the von Neumann neighbourhood considers the group of four cells in the four cardinal directions from the central one, while the Moore neighbourhood also includes the adjacent cells along diagonals. The Von Neumann neighbourhood has been chosen as the basis of the CA model developed in this work.

To give physical meaning to the rule defining water interaction between two adjacent cells, the Darcy law is assumed. According to this, the water flux between central cell and, for example, northern cell, Q_{NC} [L^3 T^{-1}], is calculated as:

$$Q_{NC} = \frac{2T_N T_C}{T_N + T_C}\left(h_N^t - h_C^t\right) \quad (1)$$

where T_N and T_C represent, respectively, the transmissivity [L^2 T^{-1}] of northern cell and central cell, h_N^t and h_C^t represent, respectively, hydraulic head [L] of northern cell and central cell at previous time step, t. The term $\frac{2T_N T_C}{T_N + T_C}$ is the harmonic mean of transmissivity. It has been chosen because of its property to remove the impacts of large outliers by limiting the flux to the lower value of transmissivity. The flux is positive if entering the central cell.

The total flux entering the central cell is (Figure 1):

$$Q_C = Q_{NC} + Q_{EC} + Q_{SC} + Q_{WC} + W_C \quad (2)$$

where W_C [L^3 T^{-1}] is the volumetric flux representing sources (+) or sinks (-).

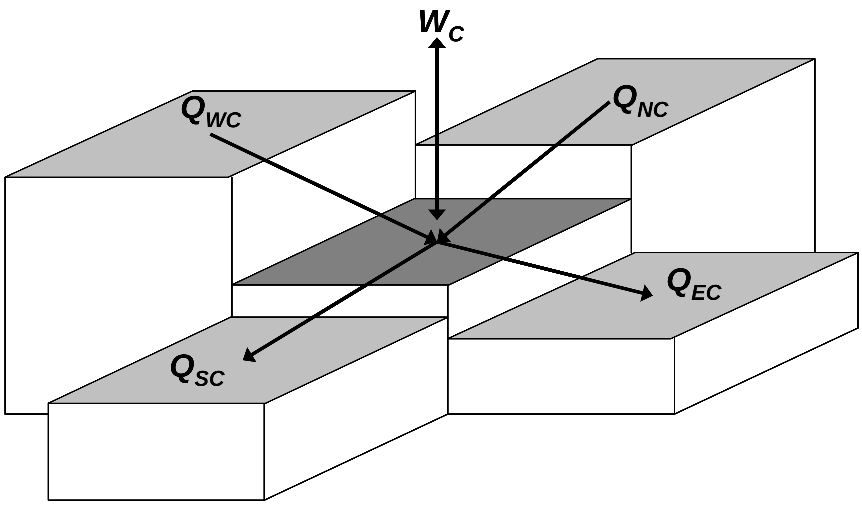

Figura 1. Scheme for the calculation of water fluxes between the central cell and the four adjacent cells. W_C is the volumetric flux representing source (entering the cell) or sink (exiting the cell).

Hydraulic head at central cell is updated for the subsequent time, *t+1*, applying the discrete mass balance equation:

$$h_C^{t+1} = h_C^t + \frac{1}{S_y}\frac{Q_C}{\Delta s^2}\Delta t \quad (3)$$

where Δt [T] is the time step.

The final component of a CA model is the boundary condition that describe what happens at the outer cells of the lattice. The boundary conditions can be of Dirichlet or

Neumann type (*Kinzelbach*, 1986). Dirichlet conditions specify the head h; Neumann conditions specify the flux, i.e., the head gradient $\partial h/\partial x$ orthogonal to the boundary. Neumann conditions are type A (permeable) or type B (impermeable). A Neumann type A condition specifies a finite gradient, i.e., $\partial h/\partial x \neq 0$; conversely, a Neumann type B condition specifies a zero gradient, i.e., $\partial h/\partial x = 0$.

3 MODEL TESTING

In order to test MACCA-GW numerical properties, a prototype artificial domain was considered, a 1-km^2 square aquifer (1 km X 1 km) with saturated hydraulic conductivity $K_s = 1.25 \cdot 10^{-5}$ m/s, and specific yield $S_y = 0.1$. The space interval was set as $\Delta s = 10$ m, i.e., a total of 100 x 100 = 10000 grid nodes. The model was subjected to two tests: the first to verify model's ability to reproduce unsteady water table depletion due to pumping from a well, and the second to test the model in an important problem in the study of alluvial aquifers that is the simulation of aquifer response to stream-stage variation. The results of MACCA-GW simulations were compared to analytical solutions, where available, and MODFLOW-2000 numerical results.

Simulations with MODFLOW-2000 were performed using harmonic mean scheme for the computation of interblock transmissivity and the WHS solver with residual tolerance for the convergence criterion = 0.0001 m that proved to be a good compromise between accuracy and computation speed.

The tests were performed on a computer with a Intel Pentium D dual core 2.80 GHz CPU and 1 GB RAM.

3.1 Drawdown due to a constant pumping rate from a well

The first stage of MACCA-GW testing has the purpose to verify the numerical model with respect to transient solution of head drawdown due to a constant pumping rate from a well. The first mathematical analysis was obtained by *Theis* (1935), under the assumptions that: (a) the aquifer is confined and compressible; (b) there is no source of recharge to aquifer; (c) water is released instantaneously from the aquifer as the head is lowered; (d) the well is fully penetrating.

The solution of unsteady distribution of drawdown is expressed by:

$$s(r,t) = \frac{Q}{4\pi T} \cdot W(u) \tag{4}$$

with

$$u = \frac{r^2 \cdot S}{4tT} \tag{5}$$

and

$$W(u) = \cdot \int_u^\infty \frac{e^{-z}}{z} \tag{6}$$

where *s*, is drawdown [L]; *Q*, is the constant pumping rate [L^3T^{-1}]; *t*, time since pumping began [T]; *r*, radial distance from the pumping well [L]. The integral expression in equation 10 is termed the well function. It is generally evaluated with analytical approximation. In this paper we adopted the solution proposed by *Barry et al*. (2000) valid for all values of the argument of exponential integral. The Theis equation can be extended to describe flow in unconfined aquifers if the drawdown is small relative to the saturated thickness of the aquifer (*Jacob*, 1950).

The domain was setup applying Dirichlet condition on the entire boundary with hydraulic head h = 50 m, as well as initial condition. A well with a constant pumping rate of 0.001 m^3/s was placed in the central cell. The time step was set to 4000 s. Monitoring wells were placed along cardinal direction at a distance of 150, 200, 300 m from the pumping well. Two monitoring wells were placed on the 45 degrees direction at a distance of 127 and 170 m to investigate the eventuality that von Neumann neighbourhood could generate privileged directions. A further monitoring well was positioned at the cell adjacent to the boundary to verify if boundary condition could have influence on the cone of depression.

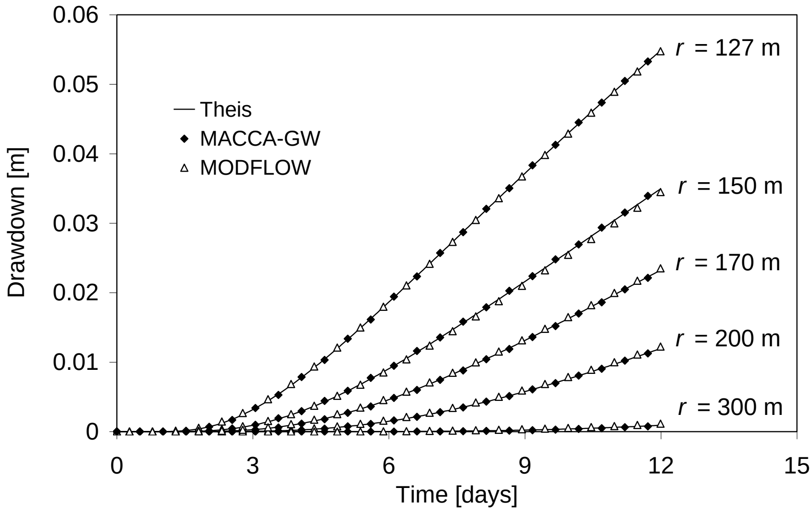

Figura 2. Comparison between analytical (Theis) and numerical solution (MACCA-GW and MODFLOW) for head drawdown due to a constant pumping rate of 0.001 m^3/s at distance r = 150, 200 and 300 m from the well along cardinal direction, and r = 127 and 170 m on the 45 degrees direction.

Figure 2 illustrates the depletion computed by MACCA-GW and MODFLOW-2000 compared to analytical solution for a 12 days duration after the beginning of the pumping. A very good fit can be observed in both monitoring wells along cardinal and diagonal direction. The calculating time was 1.125 s for MACCA-GW and 5.204 s for MODFLOW-2000.

3.2 Aquifer response to stream-stage variation

Rivers contribute water to or drain water from the ground-water system, depending on the head gradient between the river and the ground-water regime. Quantification of stream/aquifer hydraulics is an important problem in the study of alluvial aquifers.

This section has the purpose to test MACCA-GW's ability to simulate the aquifer response to stream-stage variation compared to the solution obtained by MODFLOW-2000.

The river-aquifer interconnection was simulated by use of the RIVER package in MODFLOW-2000, which allows stream to gain or lose water. The stream stage is used to calculate the flux between the stream and aquifer system, proportional to the head gradient between the river and aquifer and a streambed conductance parameter. When the aquifer head is above the bottom of the streambed, MODFLOW-2000 assumes that the discharge through the streambed is proportional to the difference in hydraulic head between the stream and aquifer:

$$Q = \frac{K_{sb} L W}{M} \left(h_w - h \right) \tag{7}$$

where Q is the discharge [L^3/T] with a downward flux assumed positive, K_{sb} is the streambed hydraulic conductivity [L/T], L is the stream length [L], W is the stream width [L], M is the streambed thickness [L], h_w is the hydraulic head in the stream [L], and h is the hydraulic head in the aquifer [L]. The term $K_{sb}W/M$ is defined hydraulic conductance of the streambed [L/T]. If the aquifer head drops below the bottom of the streambed, the model assumes that the seepage is no longer proportional to the aquifer head and becomes dependent on the water level in the stream and the streambed thickness

$$Q = \frac{K_{sb} L W}{M} \left(H_w + M \right) \tag{8}$$

where H_w is the water level in the stream above the surface of the streambed [L]. At the start of each iteration, terms representing river seepage are added to the flow equation for each cell containing a river reach.

The same scheme was implemented in the MACCA-GW model. To perform the test, the domain was setup applying a constant head h = 50 m on the west and east boundary, as well as initial condition, and a Neumann type B condition on north and south boundary. The time step was set to 4000 s. A river was placed with north-south direction at a distance of 250 m from the west boundary (Fig. 3). River bottom is supposed to be at 46.5 m. Riverbed conductivity and thickness are, respectively, $1 \cdot 10^{-5}$ m/s and 0.5 m. Width of the river is 5 m. Monitoring wells were placed at a distance of 100, 350, 450, 550, and 650 m from the west boundary as shown in Figure 3.

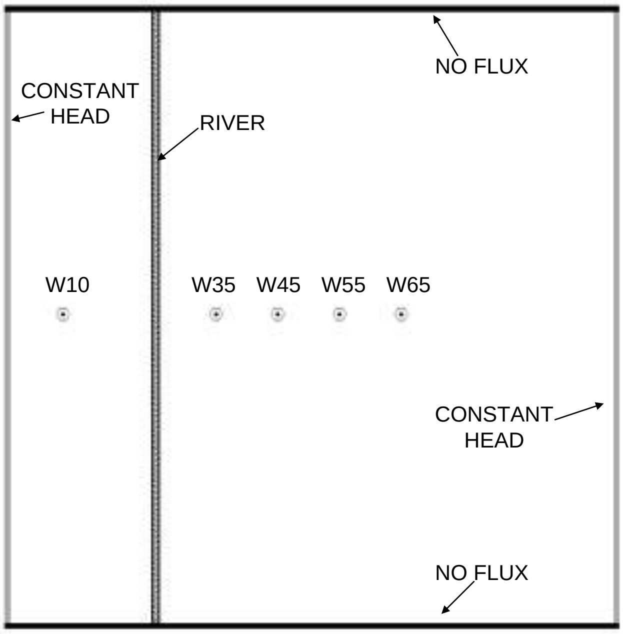

Figura 3. Scheme of the domain setup to perform the simulation of the aquifer response to stream-stage variation: location of river, boundary conditions and monitoring wells (W10, W35, W45, W55, and W65) is shown.

The simulation time was 30 days and the river stage was supposed to increase with a sinusoidal variation to a maximum of 50 m as reported in Figure 4 where the comparison between MACCA-GW and MODFLOW-2000 results is performed. A good agreement can be observed. The calculating time was 1.36 s for MACCA-GW and 16.36 s for MODFLOW-2000.

4 Conclusions

A cellular automata on a regular grid representing two-dimensional groundwater flow in unconfined aquifer was presented. Physically based equations are implemented to simulate the flow of water between adjacent cells. This makes easier the setting of model parameters and their calibration. The model can account for sources or sinks and boundary conditions of Dirichlet or Neumann type. River-aquifer interaction can be simulated: the stream stage is used to calculate the flux between the stream and aquifer system, proportional to the head gradient between the river and aquifer and a streambed conductance parameter.

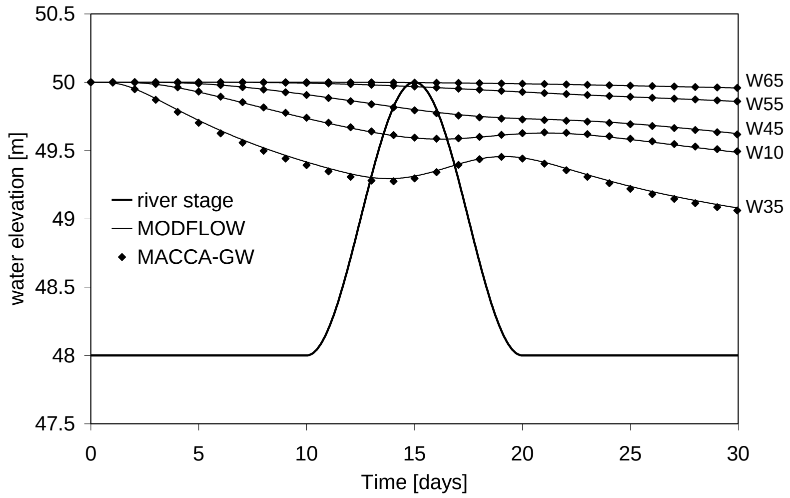

Figura 4. Comparison between analytical (Theis) and numerical solution (MACCA-GW and MODFLOW) for head drawdown due to a constant pumping rate of 0.001 m3/s at distance r = 150, 200 and 300 m from the well along cardinal direction, and r = 127 and 170 m on the 45 degrees direction.

The accuracy of the model was evaluated considering two testing problems the drawdown due to a constant pumping rate from a well, and the aquifer response to stream-stage variation. Comparison with analytical solution and MODFLOW-2000 numerical results showed a good agreement.

The MACCA-GW model, thank to the explicit numerical scheme based on macroscopic cellular automata that does not perform inner iterations, proved to be fast in simulating the investigated transient phenomena: it resulted from 4.6 to 12 times faster than MODFLOW-2000.

The code of MACCA-GW model is simple enough to facilitate its integration into other models such as distributed model that simulate water and energy fluxes at the interface between soil and atmosphere. The good performance in terms of calculating time without detriment to model's accuracy, makes the MACCA-GW adequate to perform long simulation time analysis.

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